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Distant steering: Schrödinger's version of quantum non-separability

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Abstract. Schrödinger pointed out the paradoxical fact that a skilful experimenter can steer, without any interaction, a distant particle (that is correlated with a nearby one on account of past interactions) into any of a wide set of states. He gave a sufficient condition for the range of this distant steering. In this paper a necessary and sufficient condition, enlarging Schrödinger's set in some cases, is derived. The significance of Schrödinger's approach to quantum non-separability (i.e. to quantum distant correlations) is discussed, and our previous work along these lines is put into relation with distant steering.

1. Introduction

We outline our view on two distant particles in a non-separable state, basing it on numerous real experiments performed between 1972 and 1982 (Clauser and Shimony 1978, Aspect *et al* 1982a, b). Most of these experiments were done with pairs of correlated photons emitted in a cascade de-excitation in opposite directions with distinct wavelengths λ_1 and λ_2 and with a time separation of a few nanoseconds. This was the first stage of preparation. In the second stage, one used filters (up to 15 m apart) for λ_1 and λ_2, which projected out the part of the two-photon wavefunction that corresponded to passing the filters and entering the macroscopic volumes V_1 and V_2. In each of these, an analyser and a detector (constituting together an apparatus A_1 or A_2) measured the linear polarisation of the incident photon.

The state vector of the two-photon system after the first stage of preparation factorises into a correlated (two-photon) spatial factor and a correlated (two-photon) polarisation factor χ_{12}. The second stage of preparation destroys the spatial correlation, i.e. the two-photon spatial factor further factorises. This makes the photons distant in the sense that the first is in the volume V_1 and the second in the (distant) volume V_2. At the same time, the two-photon polarisation correlation remains unchanged. As a consequence, the entire state vector ϕ_{12} (the spatial degrees of freedom of both photons included) is correlated.

In order to sum up what makes two particles *distant*, we point out that:

(i) there exist two non-overlapping spatial volumes V_1 and V_2 such that particle 1 is in V_1 and particle 2 in V_2;

(ii) the particles do not interact;

(iii) any apparatus A_1 in V_1 performing measurements on particle 1 does not interact with particle 2, and symmetrically (interchanging 1 and 2).

In all the experiments mentioned one performed coincidence measurements (with apparatuses A_1 and A_2) of polarisation observables O_1 and O_2, and simultaneously of localisation in V_1 and V_2, which is obviously compatible with polarisation.

The correlated two-photon polarisation factor χ_{12} is, in the experiments of Aspect *et al* (1982a, b), of the form:

$$|\chi_{12}\rangle = 2^{-1/2}(|x_1\rangle|x_2\rangle + |y_1\rangle|y_2\rangle)$$

where x and y refer to linear polarisation.

Quantum non-separability is implied by the validity of χ_{12} for the system at issue, and not of the corresponding separable state

$$\rho_{12} = 2^{-1}(|x_1\rangle\langle x_1|\otimes|x_2\rangle\langle x_2| + |y_1\rangle\langle y_1|\otimes|y_2\rangle\langle y_2|).$$

In other words, these two states are experimentally distinguishable. (For a historical view see appendix 1 of Clauser and Shimony (1978).)

To illustrate one way of distinguishing experimentally χ_{12} from ρ_{12}, we make use of Moldauer's Z-type observable (Moldauer 1972), which he devised for this purpose: one measures a two-photon observable that does not commute either with $|x_1\rangle\langle x_1|\otimes I_2$ or with $I_1\otimes|x_2\rangle\langle x_2|$. For example, such an observable is

$$|x_1'\rangle\langle x_1'|\otimes|x_2'\rangle\langle x_2'|$$

and the angle between x and x' is sharp. It is easy to convince oneself that this observable has different probabilities in the states χ_{12} and ρ_{12} due to interference terms.

Nowadays, when experiments have proved (Aspect *et al* 1982a, b) the reality of quantum non-separability at macroscopic distances (about 15 m), we know that local hidden variables in the sense of Einstein *et al* (1935) and Bell (1964) are discarded. We believe that Schrödinger's approach (1935, 1936) to the investigation of quantum non-separability deserves reconsideration as a research programme: a more elaborate and detailed quantum theory of distant correlations is desirable.

This paper, as well as our previous related work (see § 7), represent efforts to make a contribution along these lines.

2. Distant steering through generalised Fourier coefficients

Let $\phi_{12}\in H_1\otimes H_2$ be a pure state of two distant particles that interacted in the past, or of two distant photons that emerged from a cascade de-excitation or a positronium annihilation. (The Hilbert spaces H_1 and H_2 contain both spatial and internal degrees of freedom.)

The fact that the particles interacted in the past (or the analogous reason for photons) causes ϕ_{12} to be non-factorisable, i.e. one cannot write

$$|\phi_{12}\rangle\rangle = |\varphi\rangle|\psi\rangle \qquad |\varphi\rangle\in H_1 \qquad |\psi\rangle\in H_2.$$

But since ϕ_{12} is an element of $H_1\otimes H_2$, it equals a finite or an infinite sum of factorisable (i.e. uncorrelated) state vectors:

$$|\phi_{12}\rangle\rangle = \sum_i |\varphi_i\rangle|\psi_i\rangle.$$

This decomposition is non-unique. In the physical situation described above, it is useful to expand ϕ_{12} in an orthonormal basis $\{\varphi_i : i = 1, 2, \ldots\}$ in H_1, preferably in an eigenbasis of the observable O_1 measured by the apparatus A_1.

Lemma 1. Let $\phi_{12} \in H_1 \otimes H_2$, and let $\{\varphi_i : i = 1, 2, \ldots\}$ be an orthonormal basis of H_1. Then there exists a unique expansion

$$|\phi_{12}\rangle\!\rangle = \sum_i |\varphi_i\rangle|\psi_i\rangle \tag{1}$$

where

$$|\psi_i\rangle \overset{\text{def}}{=} \langle\varphi_i|\phi_{12}\rangle\!\rangle \qquad i = 1, 2, \ldots$$

are elements in H_2 obtained by a partial scalar product, and may be called *generalised Fourier coefficients.* If, furthermore, the vectors φ_i are eigenvectors of a complete observable O_1, i.e.

$$O_1 = \sum_i a_i |\varphi_i\rangle\langle\varphi_i| \qquad i \neq i' \Rightarrow a_i \neq a_{i'} \tag{2}$$

and if $O_1 \otimes I_2$ is measured in the state ϕ_{12}, then

(i) $\langle\psi_i|\psi_i\rangle$ is the *probability* of the result a_i,

(ii) $|\varphi_i\rangle|\psi_i\rangle/\langle\psi_i|\psi_i\rangle^{1/2}$ is the *state* of the two-particle system after the measurement has given the result a_i.

Proof. This is straightforward and can be found in Herbut and Vujičić (1976), theorem 1.

Remark 1. Statements (i) and (ii) of lemma 1 are valid even more generally: they apply to any, even to an incomplete, observable O_1 as long as φ_i is its eigenvector corresponding to a *non-degenerate* eigenvalue a_i. In particular, they apply to the projector $O_1 = |\varphi_i\rangle\langle\varphi_i|$.

From the point of view of the measuring apparatus A_1 that measures O_1, particle 1 is nearby and particle 2 is distant. Neither particle 1 nor apparatus A_1 interact with the distant particle. Nevertheless, at the end of the measurement with the result a_i the distant particle is in the state $|\psi_i\rangle/\langle\psi_i|\psi_i\rangle^{1/2}$. This is what we call, following Schrödinger (1935, 1936), *distant steering.* A skilful experimenter can, with a suitable selection of the measurement (e.g. of $O_1 = |\varphi_i\rangle\langle\varphi_i|$) on the nearby particle, steer the distant particle, with a certain probability, into the preselected states $|\psi_i\rangle/\langle\psi_i|\psi_i\rangle^{1/2}$. Varying $|\varphi_i\rangle\langle\varphi_i|$, one can thus obtain any one of a number of different states of the distant particle. One wonders how wide the set of such possible states is.

Schrödinger (1936) gave a proof that the distant particle 2 can be steered (with some probability) into any preselected state from the *range* $R(\rho_2)$ of the reduced statistical operator $\rho_2 \overset{\text{def}}{=} \mathrm{Tr}_1|\phi_{12}\rangle\langle\phi_{12}|$ by bringing the nearby particle 1 into some suitable state. Schrödinger considered this result as a genuine paradox of quantum mechanics. It was his reaction to the famous Einstein–Podolsky–Rosen (1935) argument to prove incompleteness of quantum mechanics. Schrödinger's paradox consists of the fact that the state of the distant particle depends on the selection of the observable O_1 measured on the nearby particle. This is the essential physical feature of quantum *non-separability* (d'Espagnat 1976). The paradox applies to any correlated ϕ_{12} (i.e. one that is not factorisable into states of the particles), and not only to a special class of so-called EPR states (Vujičić and Herbut 1984). We return to this point in § 7.

It is of interest to give a tightening up of Schrödinger's result.

3. Which states can be achieved by distant steering?

It follows from lemma 1(ii) that the generalised Fourier coefficient ψ_i, when normalised, is the state in which the distant particle finds itself in distant steering. The partial scalar product with the help of which ψ_i is evaluated (cf lemma 1) is in fact an antilinear operator A_a that maps H_1 into H_2 (Herbut and Vujičić 1976). Its polar factorisation gives (see (34) in the mentioned reference):

$$A_a = \rho_2^{1/2} U_a Q_1 \tag{3}$$

where ρ_2 is the reduced statistical operator describing the state of the distant particle before the steering (d'Espagnat 1976), U_a is the antiunitary correlation operator mapping the range $R(\rho_1)$ of $\rho_1 \overset{\text{def}}{=} \mathrm{Tr}_2|\phi_{12}\rangle\langle\phi_{12}|$ onto $R(\rho_2)$, and, finally, Q_1 is the range projector of ρ_1. Since

$$|\psi_i\rangle = A_a|\varphi_i\rangle = \rho_2^{1/2} U_a Q_1 |\varphi_i\rangle \tag{4}$$

it is obvious that ψ_i necessarily belongs to the range of $\rho_2^{1/2}$. What is more, we prove the following main result of this work.

Theorem. Any state vector from $R(\rho_2^{1/2})$, and only such states, can be reached by distant steering.

Proof. Let ψ be an arbitrary state vector from $R(\rho_2^{1/2})$. Then there exists a vector $\bar{\psi} \in H_2$ such that $\psi = \rho_2^{1/2}\bar{\psi}$. Denoting by Q_2 the range projector of ρ_2, one can write $\psi = \rho_2^{1/2} Q_2 \bar{\psi}$ because $\rho_2^{1/2}$ and ρ_2 have one and the same range projector (cf lemma 2 below). Next we define

$$|\varphi\rangle \overset{\text{def}}{=} U_a^{-1} Q_2 |\bar{\psi}\rangle \in H_1. \tag{5}$$

Since obviously $\langle\varphi|\varphi\rangle \neq 0$, the direct measurement of the projector $|\varphi\rangle\langle\varphi|/\langle\varphi|\varphi\rangle$ produces the state ψ by distant steering. The rest is obvious from (4).

4. Are there any states that Schrödinger did not take into account?

To see what sort of tightening we are dealing with, we prove that sometimes $R(\rho)$ is a proper subset of $R(\rho^{1/2})$.

Let ρ be any statistical operator ($0 \leq \rho$, $\mathrm{Tr}\,\rho = 1$) in a complex and separable Hilbert space H. Let

$$\rho = \sum_i r_i \sum_{m_i=1}^{M_i} |i, m_i\rangle\langle i, m_i| \qquad r_i > 0 \tag{6}$$

be a spectral form of ρ. (It should be remembered that ρ has a purely discrete spectrum, and that each of its positive eigenvalues r_i has a finite multiplicity M_i, cf Reed and Simon (1972), theorem VI.5 and VI.21.)

We make use of two linear manifolds in addition to $R(\rho)$ and $R(\rho^{1/2})$. By L we denote the set of all finite linear combinations of the eigenvectors from (6), and by $\bar{L}$ the subspace spanned by the same eigenvectors

$$\bar{L} \overset{\text{def}}{=} \left\{ \sum_i \sum_{m_i=1}^{M_i} \alpha_{i,m_i}|i, m_i\rangle : \alpha_{i,m_i} \in C, \sum_i \sum_{m_i=1}^{M_i} |\alpha_{i,m_i}|^2 < \infty \right\}.$$

Lemma 2. One has always

$$L \subseteq R(\rho) \subseteq R(\rho^{1/2}) \subseteq \bar{L}. \tag{7}$$

If ρ has no more than a *finite number* of distinct positive eigenvalues r_i, then the four linear manifolds in (7) coincide. If the number of all positive eigenvalues is *infinite*, then the four linear manifolds are all distinct:

$$L \subset R(\rho) \subset R(\rho^{1/2}) \subset \bar{L} \tag{8}$$

where '$\subset$' denotes the proper-subset relation.

Proof. Since

$$|i, m_i\rangle = \rho(r_i^{-1}|i, m_i\rangle) \qquad \forall |i, m_i\rangle$$

one has $|i, m_i\rangle \in R(\rho)$, and hence $L \subseteq R(\rho)$. Owing to

$$\rho|a\rangle = \rho^{1/2}(\rho^{1/2}|a\rangle) \in R(\rho^{1/2}) \qquad \forall |a\rangle \in H$$

one has $R(\rho) \subseteq R(\rho^{1/2})$. Finally, let $\alpha_{i,m_i} \overset{\text{def}}{=} \langle i, m_i|a\rangle$. Then, utilising the common range projector $\Sigma_i \Sigma_{m_i=1}^{M_i} |i, m_i\rangle\langle i, m_i|$ of ρ and $\rho^{1/2}$, one has:

$$\rho^{1/2}|a\rangle = \sum_i \sum_{m_i=1}^{M_i} \rho^{1/2}|i, m_i\rangle\langle i, m_i|a\rangle$$

$$= \sum_i \sum_{m_i=1}^{M_i} \alpha_{i,m_i} r_i^{1/2}|i, m_i\rangle \in \bar{L}.$$

This entails $R(\rho^{1/2}) \subseteq \bar{L}$.

In case L is finite dimensional, $L = R(\rho) = R(\rho^{1/2}) = \bar{L}$ obviously follows from $L = \bar{L}$ and (7).

Henceforth, let L be infinite dimensional. On account of $\operatorname{Tr} \rho = \Sigma_i \Sigma_{m_i} r_i = 1$, we know that $|a\rangle \overset{\text{def}}{=} \Sigma_i \Sigma_{m_i} r_i^{1/2}|i, m_i\rangle$ is an element of $\bar{L}$. On the other hand, arguing *ab contrario*, we assume that there exists a vector $|b\rangle \in H$, such that $|a\rangle = \rho^{1/2}|b\rangle$. Substituting here both

$$\rho^{1/2}|b\rangle = \sum_i \sum_{m_i} r_i^{1/2}\beta_{i,m_i}|i, m_i\rangle \qquad (\beta_{i,m_i} \overset{\text{def}}{=} \langle i, m_i|b\rangle)$$

and the above expansion of $|a\rangle$, one obtains

$$\sum_i \sum_{m_i} r_i^{1/2}|i, m_i\rangle = \sum_i \sum_{m_i} r_i^{1/2}\beta_{i,m_i}|i, m_i\rangle$$

leading to $\beta_{i,m_i} = 1$, $\forall i, m_i$. Since there is an infinite number of i, m_i values, the norm of $|b\rangle$ is infinite. Hence $|b\rangle \notin H$, $|a\rangle \notin R(\rho^{1/2})$, and hence $R(\rho^{1/2}) \neq \bar{L}$.

Taking $|a\rangle \overset{\text{def}}{=} \Sigma_i \Sigma_{m_i} r_i^{1/2}|i, m_i\rangle$ as in the preceding paragraph, we consider $\rho^{1/2}|a\rangle$ which is an element of $R(\rho^{1/2})$. We assume that $\exists |b\rangle \in H$ such that $\rho^{1/2}|a\rangle = \rho|b\rangle$. Performing scalar multiplication with $\langle i, m_i|$, we obtain $r_i\langle i, m_i|b\rangle = r_i^{1/2}r_i^{1/2}$. This again implies $\langle i, m_i|b\rangle = 1$ for all i, m_i contradicting $|b\rangle \in H$. Hence, $\rho^{1/2}|a\rangle \notin R(\rho)$, and $R(\rho) \neq R(\rho^{1/2})$.

Finally, let us renumber the pairs i, m_i as $n = 1, 2, \ldots$. We take $|c\rangle \overset{\text{def}}{=} \Sigma_{n=1}^{\infty} n^{-1}|n\rangle \in H$, and

$$\rho|c\rangle = \sum_{n=1}^{\infty} n^{-1} r_{i_n}|n\rangle \notin L$$

('i_n' is the 'i' value corresponding to 'n' in the above renumbering). Hence, $L \neq R(\rho)$.

Thus, the second inequality in (8) makes it clear that in the infinite-dimensional case there do exist states (belonging to $R(\rho^{1/2})\backslash R(\rho)$) that can be reached by distant steering and that lie outside Schrödinger's result.

If one wants to achieve in practice distant steering of particle 2 into any chosen state $|\psi\rangle \in R(\rho_2^{1/2})$ (the best one can do according to our tightened result), one can define $|\bar{\psi}\rangle \in H_2$ by $|\psi\rangle = \rho_2^{1/2}|\bar{\psi}\rangle$. Expanding

$$|\bar{\psi}\rangle = \sum_i \sum_{m_i=1}^{M_i} f_{i,m_i}|\overline{i,m}_i\rangle + |\bar{\psi}\rangle'$$

(cf (6) for $\rho = \rho_2$), $|\bar{\psi}\rangle \in R(\rho_2)^{\perp}$ (the null space of ρ_2), one can easily see that it is sufficient to bring the nearby particle 1 into the state $|\varphi\rangle/\langle\varphi|\varphi\rangle^{1/2}$, where

$$|\varphi\rangle = \sum_i \sum_{m_i=1}^{M_i} f^*_{i,m_i}|i, m_i\rangle \tag{9}$$

$|i, m_i\rangle \overset{\text{def}}{=} r_i^{-1/2}\langle\overline{i,m}_i|\phi_{12}\rangle$ and r_i satisfy (6) for $\rho = \rho_1$.

It is important to point out that distant steering of particle 2 into the state $|\psi\rangle$ need not be performed by predictive measurement of the projector $|\varphi\rangle\langle\varphi|/\langle\varphi|\varphi\rangle$. It can be done by any measurement (predictive or retrospective) of the same projector on the nearby particle (cf 6(B) in Herbut and Vujičić (1976)).

5. Can an incomplete measurement distantly steer outside $R(\rho_2^{1/2})$?

So far we have discussed distant steering assuming that one measures a ray projector $|\varphi\rangle\langle\varphi|$ on the nearby particle (cf remark 1). One wonders if distant steering can lead to some state of the distant particle *outside* $R(\rho_2^{1/2})$ as a result of a measurement of a more general projector

$$P_1 = \sum_k |\varphi_k\rangle\langle\varphi_k|$$

(e.g. an eigenprojector of an incomplete observable) on the nearby particle.

Proposition 1. The state of the distant particle after a measurement of a general projector P_1 on the nearby particle is described by a statistical operator ρ_2' that is a mixture of pure states from $R(\rho_2^{1/2})$ (cf (3)).

Proof. As is well known (Messiah 1961, p 298), the state of the two-particle system after the measurement of P_1 is

$$p^{-1/2}(P_1 \otimes I_2)|\phi_{12}\rangle$$

where $p \overset{\text{def}}{=} \langle\phi_{12}|(P_1 \otimes I_2)|\phi_{12}\rangle$. The state of the distant particle is the reduced statistical operator

$$\rho_2' \overset{\text{def}}{=} p^{-1}\,\mathrm{Tr}_1[(P_1 \otimes I_2)|\phi_{12}\rangle\langle\phi_{12}|(P_1 \otimes I_2)]$$
$$= p^{-1}\,\mathrm{Tr}_1[(P_1 \otimes I_2)|\phi_{12}\rangle\langle\phi_{12}|].$$

The last step is due to the fact that $P_1 \otimes I_2$ commutes with any operator from $H_1 \otimes H_2$ under Tr_1. Further,

$$\rho_2' = p^{-1}\,\mathrm{Tr}_1\left\{\left[\left(\sum_k |\varphi_k\rangle\langle\varphi_k|\right) \otimes I_2\right]|\phi_{12}\rangle\langle\phi_{12}|\right\}$$
$$= \sum_k (p_k/p)\rho_2'(k)$$

where $p_k \overset{\text{def}}{=} \langle\phi_{12}|(|\varphi_k\rangle\langle\varphi_k| \otimes I_2)|\phi_{12}\rangle$ and $\rho_2'(k)$ is the ray projector of the pure state

$$p_k^{-1/2}\langle\varphi_k|\phi_{12}\rangle = p_k^{-1/2}A_a|\varphi_k\rangle = p_k^{-1/2}\rho_2^{1/2}U_aQ_1|\varphi_k\rangle$$

(cf (3)).

The claim of proposition 1 is relevant also for a finite-dimensional range when $R(\rho_2^{1/2}) = R(\rho_2)$ (cf lemma 2).

6. How does the state of the distant particle change under any interaction of the nearby particle with a third system?

We assume that we have a two-particle system in the state ϕ_{12} as before. Prior to any measurement, we couple dynamically only the close particle 1 to some physical system 0 in the state $|\omega_0\rangle \in H_0$, $\langle\omega_0|\omega_0\rangle = \mathrm{Tr}_0\,|\omega_0\rangle\langle\omega_0| = 1$. One may ask how the distant particle is influenced by this coupling.

Proposition 2. The state of the distant particle changes in time by its separate evolution operator *independently* of the interaction between the nearby particle 1 and system 0.

Proof. The described dynamical coupling leads (in a certain time interval) to the following change of the state of the composite system $0+1+2$ in $H_0 \otimes H_1 \otimes H_2$:

$$|\omega_0\rangle|\phi_{12}\rangle \rightarrow (U_{01} \otimes U_2)|\omega_0\rangle|\phi_{12}\rangle.$$

The unitary evolution operator U_{01} includes arbitrary interaction between systems 0 and 1, whereas U_2 is the separate evolution operator of the distant particle. The reduced statistical operator of particle 2 in the final state is

$$\mathrm{Tr}_{01}(U_{01} \otimes U_2)(|\omega_0\rangle\langle\omega_0| \otimes |\phi_{12}\rangle\langle\phi_{12}|)(U_{01}^\dagger \otimes U_2^\dagger)$$
$$= U_2[\mathrm{Tr}_{01}(|\omega_0\rangle\langle\omega_0| \otimes |\phi_{12}\rangle\langle\phi_{12}|)]U_2^\dagger = U_2\rho_2U_2^\dagger.$$

Thus, the distant particle evolves as if there were no interaction between the nearby particle and system 0. This conclusion is valid also when system 0 is in a mixed state ρ_0 because $\mathrm{Tr}_0\rho_0 = 1$ (and this is all that is utilised in the proof).

7. Concluding remarks

Schrödinger was perplexed by the paradoxical aspects of quantum correlations. He doubted their physical reality and he suggested a mechanism for their disappearance in the framework of purely quantum mechanical ideas (cf appendix 1 in Clauser and Shimony (1978)). This so-called Schrödinger hypothesis assumed spontaneous (i.e. without measurement) wiping out of phase relations in ϕ_{12} after the spatial separation of particles 1 and 2, and conversion of ϕ_{12} into a mixture of simple product states. This hypothesis was experimentally disproved (Hooker 1972), which means that the disturbing paradox of the quantum correlations remains an unresolved problem in quantum mechanics.

Einstein *et al* (1935), on the other hand, were motivated by their belief that quantum mechanics was incomplete, i.e. that there existed local hidden variables. This gave rise to a great upsurge of theoretical and experimental research (Clauser and Shimony 1978) that ended in a complete confirmation of quantum mechanics. In the words of Peierls (1985), the situation can be compared to virtual work that confirms stability: '... we have gained confirmation of the stability of quantum theory by considering an argument which looks like upsetting it and the theory is still there'.

We expect that giving up the idea of local hidden variables should return the focus of interest to quantum mechanical aspects of distant correlations, i.e. to distant steering. We think it is necessary to analyse distant steering both theoretically and experimentally.

With this purpose in mind we have studied the most important special cases of distant steering in previous work.

(i) Distant measurement, which is distant steering in the case when the observable O_1 measured on the nearby particle is compatible with the state of this particle, i.e. $[O_1, \rho_1]=0$. In this case distant steering amounts to a measurement (of the twin observable O_2) on the distant particle (see Herbut and Vujičić 1976).

(ii) An Einstein-Podolosky-Rosen state ϕ_{12}^{EPR} allows the distant measurement of any one of two incompatible twin observables O_2 and O_2'. Such a state is characterised by the existence of at least one degenerate positive eigenvalue of ρ_1^{EPR} (see Vujičić and Herbut 1984).

(iii) The Pauli non-local correlations between two identical distant particles have been shown to be inoperative in distant correlations, i.e. to give no contribution to distant steering (cf Herbut and Vujičić 1985, 1987).

References

Aspect A, Dalibard J and Roger G 1982a *Phys. Rev. Lett.* **49** 1804
Aspect A, Grangier P and Roger G 1982b *Phys. Rev. Lett.* **49** 91
Bell J S 1964 *Physics* **1** 195
Clauser J F and Shimony A 1978 *Rep. Prog. Phys.* **41** 1881
d'Espagnat B 1976 *Conceptual Foundations of Quantum Mechanics* (London: Benjamin)
Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
Herbut F and Vujičić M 1976 *Ann. Phys., NY* **96** 382
—— 1985 *Proc. Symp. on Foundations in Modern Physics* ed P Lahti and P Mittelstaedt (Singapore: World Scientific) p 423
—— 1987 *J. Phys. A: Math. Gen.* **20** 5555
Hooker C A 1972 *Paradigms and Paradoxes* ed R G Colodny (Pittsburgh, PA: University of Pittsburgh)
Messiah A 1961 *Quantum Mechanics* vol 1 (Amsterdam: North-Holland)
Moldauer P A 1972 *Phys. Rev.* D **5** 1028

Peierls R 1985 *Proc. Symp. on Foundations in Modern Physics* ed P Lahti and P Mittelstaedt (Singapore: World Scientific) p 713
Reed M and Simon B 1972 *Methods of Modern Mathematical Physics* vol 1 (New York: Academic)
Schrödinger E 1935 *Proc. Camb. Phil. Soc.* **31** 555
—— 1936 *Proc. Camb. Phil. Soc.* **32** 446
Vujičić M and Herbut F 1984 *J. Math. Phys.* **25** 2253